

MATHEMATICIANS' SELF-CONFIDENCE AND RESPONSIBILITY

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Abstract: This article is an essay written for mathematicians. Its objective is to stimulate discussion. The essay deals with specific prejudices against Mathematics and mathematicians. Should mathematicians feel responsible to defend Mathematics against indifference or attacks? If so, when and why should we take our responsibility? Can we also speak of a mission for mathematicians? Further, are there suggestions how to assume the task of a mission? These are the questions which are the center of our discussion. Examples of rather typical situations in everyday life try to convince that such questions are important for us and our field.

1. Mathematicians and their reputation. Some people think that mathematicians are insecure, even very timid. Others, on the contrary, take them for condescending. Perhaps, as so often, the truth is somewhere in between, but I think it would be unfair to leave it like that without trying a better analysis. Can we say something more definite about shyness or self-assertion of mathematicians?

My former student Yvik Swan asked me not long ago to tell the difference between an introverted mathematician and a self-assured mathematician. I was not sure what to answer but I certainly enjoyed Yvik's description:

“We know that this is in general not true. Even more. As many independent statistical analyses show, good marks in Mathematics are the best statistical predictor of being good in all other school subjects. And if I look around among the bright Mathematics students of our University I typically see young open-minded and seemingly self-assured people. But isn't it most interesting? Even though we know that, as we have pointed out, the prejudice is in general not at all true we mathematicians still enjoy stereotype descriptions of our breed by others, as exemplified by the joke. Is it only because we really know better?”

Perhaps. Still, many people, including mathematicians, do think we should be more open, should try to communicate more precisely what we are doing and defend adequately what we are doing. Communication of what a

mathematician is doing is sometimes difficult as we know best ourselves. Nevertheless, we probably should be more willing to point out what we think whenever we believe there is a good reason to do so. It seems worth discussing a few examples.

2. Confidence in Mathematics. When a medical doctor whom we meet for the first time tells us after two minutes that he or she was always a zero in Mathematics, I think we should not say that we have heard this from others. Still worse would be adding that we know others who were very bad in Mathematics and succeeded well in their life. I am sure that a better answer is “Oh, this is very sad! This is likely to impair all those of your decisions which involve elementary logic or a sound feeling for the order of magnitude of numbers.” Elementary logic is a must for medical doctors, and elementary logic is one of the typical and far-bearing failures of doctors who say that they were very bad in Mathematics. The immediate reaction of the surprised doctor on our alert is to respond that they are of course “very logical”, simply trying to say they could not do the other stuff with x ’s or $f(x)$ ’s and so on. Realizing that you doubt his or her “logical abilities”, he or she may be annoyed by now. However, we should firmly show how much we worry about what we just heard and should not compromise. As politely as we can, we should insist on our sadness by telling the further truth: “Unfortunately, the lack of knowing a minimum of how to handle x ’s or $f(x)$ ’s or so at the end of high school is known to be highly correlated throughout life with failures in elementary logic and a bad feeling for orders of magnitudes.”

A physician who cannot negate correctly a statement containing one or two implications or one or two and/or attributes can do true harm to a patient. The same holds for the failure to see or to use (our beloved) equivalence of « A implies B » and « not- B implies not- A . » Mathematicians use the latter of course mostly in the convenient form « A and not- B implies a contradiction » for a proof by contradiction. This appealing form is however equally important for everyday decisions. Just as a test mind. Indeed, many wrong statements, worldwide, and in many domains, could probably be avoided by applying this equivalence on a regular basis. There are several other examples of elementary reasoning of similar importance. Unfortunately, the importance of elementary logic is seemingly not that clear to everybody. Otherwise textbooks of general Mathematics for medicine, biology or pharmacy students for instance would all have a dominant introduction to elementary reasoning and its importance, but this is far from being the case.

Another typical and sometimes dangerous error lies is on the side of elementary probabilistic or statistical reasoning, in particular reasoning

involving conditional probabilities, the regression effect, Simpson's paradox, and so on. The example of 82 percent wrong answers of medical doctors and students in a test of Bayes' formula at Harvard Medical School – arguably one of the most prestigious medical schools in the world – shows how serious the problem can be (see [1]). The largest block of wrong answers were even drastically wrong with estimating the probability of the incidence of the tested disease to be 95 percent whereas the correct answer was close to 2 percent (see Casscells et al., [1], 1978). This is some time ago, but we should not be mistaken. Things have not necessarily improved (see for instance [2], 2009). According to my own modest experience, it is also linked with a lack of interest. When discussing more recently the results of a multi-targeted blood test I dared to remark that if a test goes over a sufficiently large number of factors which are seen as relatively independent of each other then it becomes “not-normal” to have everywhere “normal” values.

It's no problem of course, that the physician does not see immediately what we mean. The problem begins when we see that he or she is not interested in understanding what we may have possibly meant. Similarly, pretending to analyze data of a test and proposing medication having no idea whether the order of magnitude of the standard deviation is 0.1, or rather 1, or even 5, and not being keen to look it up, this is more than a *faux pas*; it is a lack of responsibility. We should not accept this; we should react and take our responsibility. If we leave the doctor's cabinet politely on basis of such a conversation, he or she will remember, and possibly for good. This is probably a wise choice for us; in the same vein, it may be for the benefit of society and undoubtedly helpful for the appreciation or recognition of our field. In summary, I dare to affirm my conviction that no doctor who advertises his mathematical incompetence, however competent he/she may be for certain problems, should be trusted in medical decisions involving weighing alternatives, and that it is our and mainly our responsibility to point this out.

It would be inadequate and unfair to exemplify medical doctors without mentioning other examples. So I remember a round-table discussion in Germany with a member of the European Commission as the main speaker. This was shortly before the first introduction of the Euro-currency (around 2000). He spoke about the laws and recommended rules which would be implemented to make the European Central Bank an institution at least as reliable as the Deutsche Bundesbank, and hence, he argued, the Euro at least as strong as a currency as the Deutsche Mark. He reasoned “if you increase the size of an Institution by a factor c you must increase control by factor c^2 that the European Union has (of course) the financial means to do this. I was

not sure how an increase of control was supposed to be measured, quite apart from the important fact that now member states with national interests would all have their word to say in this new central bank. In particular I wondered why he could assume such a simple linear relationship between control and safety. I must admit that my question was hardly helpful for the later discussion but I am still glad that I had tried to draw attention to conclusions which seemed naive.

Why is it so that people have difficulties to listen to us even if we just try to say elementary and useful things? The problem seems to be rather widespread. Brian Vidakovic (Atlanta) commented “Doctors and lawyers are the typical bad guys but we get it also from the unlikely sources that should appreciate Mathematics better: engineers, biologists etc.” Is it some impression we may leave on others? Too modest, insecure, no humor, too proud, condescending? Let us have a short look at this.

3. Modesty, Condescendence and Contradictions. Can mathematicians be proud, even condescending? Well, we can imagine John Littlewood would have said there is “hope”. As far as I remember the following examples, among my preferred ones, can be found in Littlewood’s Miscellany [3]:

Two dons from the Mathematics Department of Oxford or Cambridge at high table: “Did you hear already that our colleague such and such is to become the new Minister of Defence?”

- “Really? Oh, what a come-down!”

“What did God do before creation?”

- “Thousand pages must have been written on this; but he was doing Pure Mathematics and thought it would be a nice change to do some Applied.”

There are certainly many mathematicians who have a superb humor. The eminent Carl Friedrich Gauss, known for so many wonderful things but hardly cited for his humor, makes no exception (see for instance the introduction in [4]). And as mathematicians we should be proud of this witty side of so many of us. A few mathematicians, like anybody else in any arbitrary field, may, of course, be condescending in some way or another. But I think we hardly see a problem which would be specific for mathematicians. In particular we do not know the notion of a “school” in Mathematics as understood in many other fields. This helps to keep standards of honesty and prevent what one may call artificial condescendence. Even the beloved picking order Pure Mathematics Applied Mathematics is for us rather a cultivated recurrent amusement than a real problem.

Sometimes it is almost the contrary of pride or condescendence which can be a problem, however. Even the most brilliant of our breed, those who would have all reason to stand up in the world, to stand up for Mathematics, do not always do so in an inadequate form. I remember a reader of the Newsletter of the EMS of some years ago who complained in his letter that one of the awardees of one of our most distinguished prizes in Mathematics showed up at an official event dressed very “unconventionally”, if we may say so, and behaving accordingly. Fortunately I did not see this and don’t know to which extent it was true in the sense that many of our colleagues would have felt the same. However, if it was like that, we should be grateful for the complaint and encourage similar letters in similar cases. Perhaps these letters would imply second thoughts. Inadequate dress or behavior is likely to harm our field, I believe. If the celebrity cannot be suspected of the slightest bad intention then this is a different matter but the effect stays still the same.

We cannot prove mathematically that the lack of respect for conventions is likely to harm, but we may try an analogy. Let us visualize in our mind the portraits of Gauss, Euler, Newton, Leibniz, the Bernoullis, Laplace, Quetelet or others of our celebrities, just as we know them. They appear before us dressed in the style a person in their position would dress at his time for a portrait or an official event. Would we like to see them differently? Even if we are indifferent, would we expect our former heroes to not have been as brilliant as the new one to dare to break conventions? (NB: Heroes, like everybody else, are allowed to break conventions but should do the thinking before and then be consistent. - Respect for Perelman.)

Now to another problem, smaller but still of some concern. This is an exaggerated display of modesty. Mathematics is, I believe, no place for the strategy to “stoop to conquer”. If you name your textbook “Elements of ...” it is arguably not a good idea to turn off the reader by a first page which most people do not understand! Similarly, if you have excused yourself already twice in your Introduction that your presentation will lack mathematical rigor, how consistent is then to refer to Bourbaki twice on the first page? One should keep in mind that the reader of a book may have

because he saw no alternative to beginning with “Elements”. He will now feel badly served.

Eminent mathematicians should take particular care if they want to serve our field, or help others to serve our field. Exaggerated modesty towards the public seems awkward. If a Fields medalist, say, expresses amazement that he or she is invited to speak on aspects of Mathematics at a most prestigious occasion, underlining repeatedly that many others would be

more competent to do so, then this has two sides of the coin. It may be alienating rather than polite. Come on, Professor, the audience may think, you got the Fields Medals in Mathematics, you are not invited because you won the second prize in raising white rabbits. Don't you mathematicians say yourself that this medal is the highest distinction for young Mathematicians and comparable to the Nobel Prize? - Hic Rhodos, hic salta!.

Also, all of us, eminent or not eminent, make errors in judgment, errors in prediction. This needs no explanation. But we try to do our best saying things we can prove, and if not, in which we believe. Is it then suitable to advertise that one probably will change one's point of view in five years completely? With all respect, one should see the implication: A listener may think, this means that my decision to come here today was probably a completely bad investment.

To summarize, Mathematics is for many people a mysterious subject, possibly a suspicious subject. However, it is undoubtedly seen as a difficult discipline and not a ridiculous one. Any public with self-respect and a minimum of interest in our field expects to be treated correspondingly. In accordance with this awareness the audience has a right: Entitled to admire but equally entitled to be treated with respect. This is why mathematicians, whatever their level, whatever their fame, have a responsibility towards the field of Mathematics, be it in front of an audience, or somewhere else.

If the situation requires, the feeling for responsibility should turn into a mission.

4. Responsibility and Mission. The challenge of responsibility may arise everywhere and any day. The last example is intended to show that the charge or challenge of a mission may have to be assumed already on a very basic level.

You happen to sit with four lawyers, or lobbyists having studied law, on the same reception table (such a constellation is, by the way, not unlikely in certain towns as for instance Brussels). It is sad enough if two of these not only praise their complete incompetence in Mathematics but also try to ridicule our subject. We are put on test, but things can become worse. The third man of Law says he cannot imagine that Mathematics has any impact on real life decisions. The fourth shows no support either. Grateful enough,

The evening threatens to become rather short, but the mission begins. We may try to change the subject. Why not Law? We show interest. No need to faint it, we mathematicians are interested, so we ask questions. With four

layers on your table we are likely to hear from one of them, early enough, that lawyers are superior in logical thinking to anybody else, simply by the great importance the study of Law attaches to consistency and the frequent use of logical arguments in general. But we may confidently take the risk: “Why do you believe this? I am convinced that nobody of you would have a chance to compete in logic with an average colleague of mine, simply by the permanent requirement of all this in Mathematics”. (It is true that some people in law are really very good, and I beg their pardon.)

Sure enough we can live for half an hour with two miffed faces at our table annoyed by our unexpected arrogance. Of course we were not arrogant at all, we told the truth. Moreover, there is a good chance that we have made a point for Mathematics. We may win over one of the lawyers on the table, perhaps almost two, at least in the sense they admit that we also may have a point, somewhere..., somehow.... Now, having a lawyer half on our side, things become easier. Another one may follow to some extent, and the evening may turn out not too bad after all.

Clearly, I have presented here bad-case or worst-case situations, though no invented scenarios. Good doctors, competent economists and well-educated lawyers will forgive me because they know that we know that there are many really good ones of them. Some others may go so far to say: no need to excuse. Other readers may have again experiences of a different kind. Whatever they may be, if they challenge Mathematics, if they require to take our responsibility, let us stand up!

5. Conclusion. The reader may find that I am somewhat too sensitive with respect to the outside world view of Mathematics, and its implications. He may be right. It may also have become stronger over the years, possibly favored by challenges of the entourage of a town like Brussels. Am I opinionated? I am, and I am afraid I should be as much as those respected colleagues who confidently play on the other side, that is, who assume the role of apostles preaching that Mathematics in all its supremacy simply does not have to care, that no one in Mathematics should be obliged to show

I respect their feelings, but it is clear from what I said that I think they are wrong.

This does not imply that I am right. But would you write such an article if you are not convinced that you are most probably right? Hence it may be a good idea to discuss. And those of us who are engaged in some work of public awareness may want to examine whether our efforts to raise the public awareness and the image of Mathematics are sufficiently effective.

Self-assertion, whenever justified, respect for the public, whenever expected, and courage civil, whenever needed, might, as I argue, add a lot to their effectiveness.

In contrast to the style presented here many colleagues may agree with the essence of what is said in this article. Hence it may offer nothing really new to them. If so, it would be no surprise if they advised me to do more Mathematics and not to spend time with such an article. -- I must admit I should really try my very best to follow their first advice.

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